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THE EFFECTS OF INTERFERNCE ON MONOPULSE PERFORMANCE

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16. Abstract

The first and second moments of the monopulse azimuth estimates are used to characterize the estimator performance in a background of mainbeam or side-lobe multipath or ATCRBS interference. An exact expression for the bias error is obtained that not only accounts for the bias due to targets separated in azimuth but also demonstrates the effect of fading or signal cancellation. For a target signal at a 20 dB signal-to-noise ratio, it is shown that the fading effect produces negligible bias unless the signal-to-interference ratio is between ±2.5 dB. A first order expression for the variance is obtained that demonstrates that an increase in the random error will occur due to the target separation in azimuth and due to signal fades. The exact nature of the bias and variance depend strongly on the relative phase between the target and interference signals with the poorest performance being obtained at the in- and out-of-phase conditions. It is knowledge of this behaviour that is essential in evaluating the idea of azimuth estimation data editing.

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CONTENTS

List of Illustrations		iv
1.	Introduction	1
2.	Interference Effects on Estimate Bias	2
3.	Special Case: Real Antenna Patterns	8
4.	Interference Effects on Estimate Variance	21
5.	Conclusions	27
References		29
Appendix		30
Acknowledgment		31

LIST OF ILLUSTRATIONS

Figure		Page
1(a)	EQUIVALENT AZIMUTH USING THE MONOPULSE FUNCTION	12
1(b)	PROBABILITY DISTRIBUTION FUNCTION OF EQUIVALENT AZIMUTH	12
2.	AZIMUTH ESTIMATE IN THE PRESENCE OF INTERFERENCE	1[illegible]
3.	AZIMUTH ESTIMATE IN THE PRESENCE OF INTERFERENCE	18
4.	VARIANCE OF THE AZIMUTH ESTIMATE IN THE PRESENCE OF INTERFERENCE	26

1. INTRODUCTION

There are several competing factors that limit the performance of the monopulse azimuth estimation accuracy in a noise and interference background. The first is due to the fading phenomenon that results in a degradation in the signal-to-noise ratio (SNR) as the target and interference signals approach the out of phase condition. Pruslin [1], in a simulation of the receiver noise effects on monopulse performance, has observed that the estimator becomes biased for low values of the SNR. Browne [2], in an interference-free analysis, has calculated this bias exactly for all values of the SNR.

Closely related to the bias introduced by fading, is what we shall refer to as the azimuthal bias that results when the interferer and the target signal sources are located at different azimuths. Finally, the presence of interfering signals causes the variance of the estimate to increase.

In this note we shall study mainbeam and sidelobe interference and noise effects simultaneously and use an analysis similar to Browne's to compute an exact expression for the estimator bias. We specialize our results to real antenna patterns and obtain an expression for the bias that holds for the high and low SNR situations, hence, we obtain the azimuthal bias and the fading bias simultaneously. Numerical results are given for some typical operating conditions and it is shown that the bias can be a large fraction of a beamwidth.

Some further analytical results are obtained for the case of sidelobe interference for real antenna patterns. In this case too, it is possible for an ATCRBS (Air Traffic Control Radar Beacon System) interferer that is located in a large near-in sidelobe to cause bias errors that are a large fraction of a beamwidth.

In addition to introducing a deterministic bias to the azimuth estimate, interference causes the random error to have an increased variance. We obtain a general expression for the variance that applies to the mainbeam and sidelobe interference cases using an analysis similar to that developed by Sharenson [3] who analyzed the noise-only case. Numerical results are given that show that the variance also depends critically on the relative phase between the target and interference signals.

These general expressions for the bias and the variance of the monopulse estimate provide the tools needed to thoroughly understand the effects of interference. These are essential in the evaluation of signal processing techniques to overcome the effects of interference. The leading candidate at the moment, is the monopulse interference detector and data editing scheme followed by an outlier test. Although this is the subject of a separate paper [4] the results of the present study are the background for the succeeding analysis.

2. INTERFERENCE EFFECTS ON ESTIMATE BIAS

We restrict our attention to sum-difference (even-odd) monopulse processing that is performed on a sampled-data basis. Assuming mixer preamplifiers at the output of the sum and difference beams, the received signals in the presence of interference are modelled by

$$y_i = A_s e^{j\varphi_s} G_i(\theta_s) + A_I e^{j\varphi_I} G_i(\theta_I) + n_i \qquad i = 1, 2 \tag{1}$$

where y_1, y_2 refer to the outputs of the sum and difference beams; A_s, φ_s, θ_s and A_I, φ_I, θ_I are the amplitude, phase, and azimuth of the target and interference respectively; $G_1(\ )$, $G_2(\ )$ are the antenna patterns of the sum and difference beams, and these may be complex in general; n_1, n_2 are the independent, zero mean Gaussian noise samples due to the mixer preamps whose real and imaginary parts have variance σ^2.

An azimuth estimator that is often used in practice is given by the relation

$$E(\hat{\theta}) = \mathrm{Re}(y_2/y_1) \tag{2}$$

where the monopulse function, $E(\theta)$, is simply the normalized difference pattern,

$$E(\theta) = G_2(\theta)/G_1(\theta) \tag{3}$$

In most cases of practical interest, this function is well approximated by a linear characteristic, hence we can write

$$E(\theta) = k(\theta/\theta_B) \tag{4}$$

where θ_B refers to the 3 dB beamwidth of the antenna patterns and k is a standard parameter that arises in the characterization of monopulse systems.

Usually $1 \leq k \leq 2$ with $k = 1.5$ being a typical value. When this approximation is valid, the azimuth estimate is readily generated as

$$\frac{\theta}{\theta_B} = \frac{1}{k} \operatorname{Re}\left(\frac{y_2}{y_1}\right) \tag{5}$$

We will incorporate θ_B into our definition of θ and hence express all our results in units of a 3 dB beamwidth.

Our goal is to compute the mean and variance of (5) when interference is present in the sum and difference signals as given by (1). We begin by letting

$$\mu_i = A_s G_i(\theta_s) e^{j\varphi_s} + A_I G_i(\theta_I) e^{j\varphi_I} = V_i e^{j\beta i} \tag{6}$$

Since n_i are complex Gaussian variates, they can be expressed as $n_i = N_i \exp j\alpha_i'$ where N_i are Rayleigh and α_i' are uniform random variables. Using these definitions in (1) we have

$$y_i = V_i e^{j\beta_i} + N_i e^{j\alpha_i'} = e^{j\beta_1} \left[V_i e^{j(\beta_i - \beta_1)} + N_i e^{j(\alpha_i' - \beta_1)} \right] \tag{7}$$

Since α_1' and α_2' are independent and uniform, then so too are $\alpha_1' - \beta_1 \triangleq \alpha_1$. We apply these relations to evaluate (5) by writing

$$\mathrm{Re}\left(\frac{y_2}{y_1}\right) = \frac{\mathrm{Re}(y_2 y_1^*)}{|y_1|^2}$$

$$= \frac{\left[V_2\cos(\beta_2 - \beta_1) + N_2\cos\alpha_2\right]\left(V_1 + N_1\cos\alpha_1\right) + \left[V_2\sin(\beta_2 - \beta_1) + N_2\sin\alpha_2\right]N_1\sin\alpha_1}{V_1^2 + 2V_1N_1\cos\alpha_1 + N_1^2} \tag{8}$$

Following Browne [2] we first average (8) over α_2. Since this is uniform on $(0, 2\pi)$, we have

$$E_{\alpha_2}(k\hat{\theta}) = \frac{V_2\cos(\beta_2 - \beta_1)\,(V_1 + N_1\cos\alpha_1) + V_2N_1\sin(\beta_2 - \beta_1)\sin\alpha_1}{V_1^2 + N_1^2 + 2V_1N_1\cos\alpha_1} \tag{9}$$

Next he averages over α_1. In the Appendix this is shown to give

$$E_{\alpha_1\alpha_2}(k\hat{\theta}) = \begin{cases} 0 & \text{if } N_1 > V_1 \\ \dfrac{V_2}{V_1}\cos(\beta_2 - \beta_1) & \text{if } N_1 < V_1 \end{cases} \tag{10}$$

Since N_1 is the magnitude of a complex Gaussian random variable having variance $2\sigma^2$, it has the Rayleigh distribution

$$p(N_1) = \frac{N_1^2}{\sqrt{2}\sigma} \exp\left(-\frac{N_1^2}{4\sigma^2}\right) \qquad \text{for } N_1 \geq 0 \tag{11}$$

Then averaging (10) over N_1 gives

$$E(\kappa\theta) = \frac{V_2}{V_1} \cos(\beta_2 - \beta_1) \frac{1}{\sqrt{2}\sigma} \int_0^{V_1} N_1 \exp\left(\frac{-N_1^2}{4\sigma^2}\right) dN_1$$

$$= \frac{V_2}{V_1} \cos(\beta_2 - \beta_1) \left[1 - \exp\left(\frac{-V_1^2}{2\sigma^2}\right)\right] \tag{12}$$

It should be noted that when interference is absent, $A_I = 0$ and from (6) we see that $\beta_1 = \beta_2$ and (12) reduces to the same expression obtained by Browne. However, we are now in a position to determine the effects of interference on the azimuth bias. To do this we need to evaluate V_1, V_2/V_1 and $\beta_2 - \beta_1$ from (6).

Before performing this evaluation we first note that the target of interest lies within the mainbeam of the antenna, hence we may assume that $G_1(\theta_s)$ and $G_2(\theta_s)$ are real functions of θ_s. Whereas the sum and difference beams are in phase over the antenna beamwidth, small errors in the amplitude and phase tapers render them complex in the sidelobes. Since the interference can be located in the mainbeam or within the sidelobes of the antenna, then $G_i(\theta_I)$ must be considered to be complex in general. To make this explicit we write

$$G_i(\theta_I) = A_i(\theta_I) \exp \Psi_i(\theta_I) \tag{13}$$

Then substituting this into (6) we can easily show that

$$V_i = \left\{ A_s^2 G_i^2(\theta_s) + 2A_s A_I G_i(\theta_s) A_i(\theta_I) \cos\left[\varphi + \Psi_i(\theta_I)\right] + A_I^2 A_i^2(\theta_I) \right\}^{1/2} \tag{14}$$

$$\theta_i = \varphi_s + \tan^{-1} \left\{ \frac{A_I A_i(\theta_I) \sin\left[\varphi + \Psi_i(\theta_I)\right]}{A_s G_i(\theta_s) + A_I A_i(\theta_I) \cos\left[\varphi + \Psi_i(\theta_I)\right]} \right\} \tag{15}$$

where $\varphi = \varphi_I - \varphi_s$ is the relative phase between the target and interference signals. Using these expressions in (12) and performing straightforward but tedious trigonometric manipulations we can show that the first moment of the azimuth estimate is

$$\bar{\hat{\theta}} - \bar{\hat{\theta}}_o \left\{ 1 - \exp\left\{ - \frac{A_s^2 G_1^2(\theta_s)}{2\sigma^2} \left[1 + 2\rho \frac{A_1(\theta_I)}{G_1(\theta_s)} \cos\left[\varphi + \Psi_2(\theta_I)\right] + \rho^2 \frac{A_1^2(\theta_I)}{G_1^2(\theta_s)} \right] \right\} \right\} \tag{16}$$

where $\rho = A_I/A_s$ represents the interference-to-signal ratio (ISR) prior to any processing and

$$k\bar{\hat{\theta}}_o =$$

$$\frac{\dfrac{G_2(\theta_s)}{G_1(\theta_s)} + \rho^2 \dfrac{A_2}{G_1}\dfrac{A_1}{G_1}\cos(\Psi_2 - \Psi_1) + \rho\left[\dfrac{G_2^2(\theta_s)}{G_1^2(\theta_s)}\dfrac{A_1^2}{G_1^2} + \dfrac{A_2^2}{G_1^2} + 2\dfrac{G_2(\theta_s)}{G_1(\theta_s)}\dfrac{A_2}{G_1}\dfrac{A_1}{G_1}\cos(\Psi_2 - \Psi_1)\right]^{1/2}\cos(\varphi + \gamma)}{1 + 2\rho\dfrac{A_1}{G_1}\cos(\varphi + \Psi_1) + \rho^2\dfrac{A_1^2}{G_1^2}} \tag{17}$$

where

$$\gamma = \tan^{-1}\left[\frac{G_2(\theta_s)\, A_1 \sin\Psi_1 + G_1(\theta_s)\, A_2 \sin\Psi_2}{G_2(\theta_s)\, A_1 \cos\Psi_1 + G_1(\theta_s)\, A_2 \cos\Psi_2}\right] \tag{18}$$

These are the key expressions we need to fully understand the deterministic aspect of the azimuth error introduced by interference.

3. SPECIAL CASE: REAL ANTENNA PATTERNS

The results that have been obtained to this point are free of any approximating assumptions and apply to the general case of complex antenna patterns. Unfortunately it is difficult to draw analytical conclusions from these expressions because of the large number of parameters that would have to be examined simultaneously. Results for the general case will be obtained using a computer simulation. There is one case of considerable practical importance that we can explore analytically. This is the situation in which the antenna patterns are real, which is a reasonably accurate model for the high near-in sidelobes of the antenna. For this case we assume that

$$A_i(\theta_I)\sin\Psi_i(\theta_I) \approx 0 \qquad i = 1,\ 2 \tag{19}$$

and then define

$$G_i(\theta_I) = A_i(\theta_I)\cos\Psi_i(\theta_I) \tag{20}$$

which we note are real functions for all values of θ_I. Using this assumption, (16) and (17) become

$$\bar{\hat{\theta}} = \hat{\theta}_o\left\{1 - \exp\left\{-\frac{A_s^2 G_1^2(\theta_s)}{2\sigma^2}\left[1 + 2\rho\frac{G_1(\theta_I)}{G_1(\theta_s)}\cos\varphi + \rho^2\frac{G_1^2(\theta_I)}{G_1^2(\theta_s)}\right]\right\}\right\} \tag{21}$$

where now

$$k\hat{\theta}_o = \frac{\frac{G_2(\theta_s)}{G_1(\theta_s)} + \rho\left[\frac{G_2(\theta_s)}{G_1(\theta_s)}\frac{G_1(\theta_I)}{G_1(\theta_s)} + \frac{G_2(\theta_I)}{G_1(\theta_s)}\right]\cos\varphi + \rho^2\frac{G_2(\theta_I)}{G_1(\theta_s)}\frac{G_1(\theta_I)}{G_1(\theta_s)}}{1 + 2\rho\frac{G_1(\theta_I)}{G_1(\theta_s)}\cos\varphi + \rho^2\frac{G_1^2(\theta_I)}{G_1^2(\theta_s)}} \tag{22}$$

We recall that $\rho = A_I/A_s$ represents the interference-to-signal ratio (ISR) prior to any processing. The measured ISR at the outputs of the antenna ports is given by

$$\rho_o = \frac{A_I G_1(\theta_I)}{A_s G_1(\theta_s)} = \rho \frac{G_1(\theta_I)}{G_1(\theta_s)} \tag{23}$$

Although both definitions of ISR are important parameters of the system, it is convenient to rewrite (21) and (22) in terms of ρ_o. In this case the mean value of the azimuth estimate is given by

$$\bar{\hat{\theta}} = \bar{\hat{\theta}}_o \left\{ 1 - \exp\left[\frac{-A_s^2 G_1^2(\theta_s)}{2\sigma^2} \left(1 + 2\rho_o \cos\varphi + \rho_o^2\right) \right] \right\} \tag{24}$$

where now

$$k\bar{\hat{\theta}}_o = \frac{\dfrac{G_2(\theta_s)}{G_1(\theta_s)} + \rho_o \left[\dfrac{G_2(\theta_s)}{G_1(\theta_s)} + \dfrac{G_2(\theta_I)}{G_1(\theta_I)} \right] \cos\varphi + \rho_o^2 \dfrac{G_2(\theta_I)}{G_1(\theta_I)}}{1 + 2\rho_o \cos\varphi + \rho_o^2} \tag{25}$$

It is interesting that the performance depends only on the monopulse function $E(\theta) = G_2(\theta)/G_1(\theta)$. In most cases of interest the target will be located within the 3 dB beamwidth of the antenna. Therefore, using (3) and (4) we can write

$$\frac{G_2(\theta_s)}{G_1(\theta_s)} = k\theta_s \tag{26}$$

where θ_s is measured in 3 dB beamwidths. In the case of mainbeam interference this linearity property will also be valid, but in more general cases $E(\theta_I)$ will simply take on the values of the normalized difference pattern. To handle this situation we define an equivalent azimuth

$$\tilde{\theta}_I = \frac{1}{k} \frac{G_2(\theta_I)}{G_1(\theta_I)} \tag{27}$$

and note that whenever θ_I is within the 3 dB beamwidth of the antenna $\tilde{\theta}_I \approx \theta_I$. In Figure 1(a) we have plotted the equivalent azimuth for a monopulse antenna configuration that has a 4° beamwidth and achieves a -20 dB peak sidelobe level on both the "sum" and "difference" beams. Assuming that θ_I is uniformly distributed in $(-\pi, \pi)$, Figure 1(b) shows the probability distribution function of the equivalent azimuth. This shows that "most of the time" the equivalent azimuth will be less than 2 beamwidths, hence, for the purpose of analysis, we can study the cases of mainbeam and sidelobe interference simultaneously. Then using (26) and (27) in (25) we can express the first moment of the azimuth estimate as

$$\bar{\hat{\theta}} = \bar{\hat{\theta}}_o \left\{ 1 - \exp\left[- \frac{A_s^2 G_1^2(\theta_s)}{2\sigma^2} \left(1 + 2\rho_o \cos\varphi + \rho_o^2\right) \right] \right\} \tag{28}$$

$$\bar{\hat{\theta}}_o = \frac{\theta_s + \rho_o(\theta_s + \tilde{\theta}_I)\cos\varphi + \rho_o^2 \tilde{\theta}_I}{1 + 2\rho_o \cos\varphi + \rho_o^2} \tag{29}$$

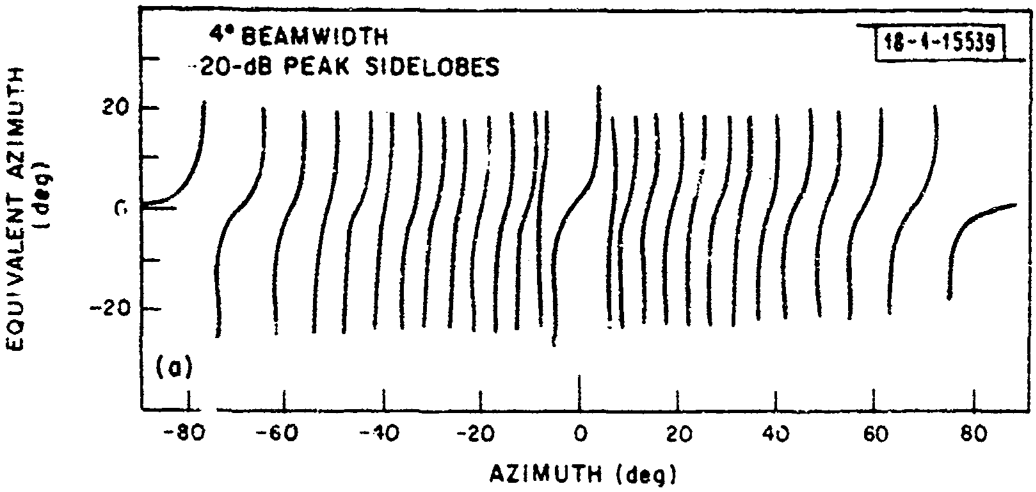

Fig. 1a. Equivalent azimuth using the monopulse function.

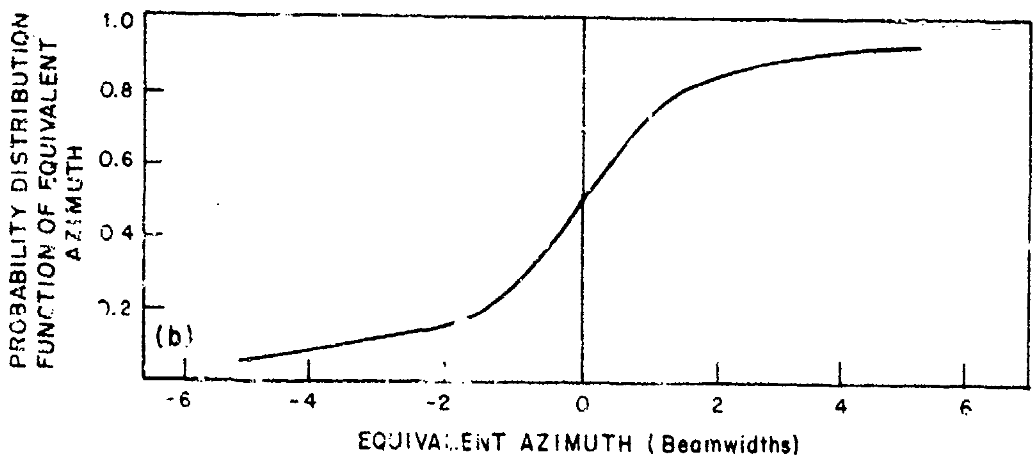

Fig. 1b. Probability distribution function of equivalent azimuth.

It is from this equation that we can begin to make some general statements regarding the performance of the azimuth estimator. We shall first show how the results are consistent with those previously obtained and then go on to consider cases that have not yet been explored in the literature.

(a) Case 1: No Interference

When there is no interference, $A_I = 0$, hence $\rho_o = 0$ and from (28) and (29) the average value of the estimate is

$$\bar{\hat{\theta}} = \theta_s \left\{ 1 - \exp\left[- \frac{A_s^2 G_1^2(\theta_s)}{2\sigma^2} \right] \right\} \tag{30}$$

This is the result obtained by Browne [2] and describes analytically Pruslin's observation [1] that as the SNR, $A_s^2 G_1^2(\theta_s)/2\sigma^2$, decreases, the monopulse estimate is biased towards the antenna boresight.

We have tabulated (30) in Table 1 from which it can be seen that for SNR's above 8 dB, the bias can be considered to be negligible. Since in the Air Traffic Control (ATC) system the SNR will be at least 20 dB, this effect will be insignificant unless fading occurs, as will be discussed in the next section.

Table 1. Variation of Bias with SNR

SNR (dB)	$\frac{\bar{\hat{\theta}} - \theta_s}{\theta_B}$
0	0.37
3	0.135
3.6	0.1
4.76	0.05
6.6	0.01
7.25	0.005
8.39	0.001
8.8	0.0005
9.64	0.0001

(b) Case 2: Fading

If the target and interference signals arrive from the same azimuth (multipath from a flat earth for example) then $\tilde{\theta}_I = \theta_I = \theta_S$ and (28) and (29) reduce to

$$\bar{\hat{\theta}} = \theta_s \left\{ 1 - \exp\left[- \frac{A_s^2 G_1^2(\theta_s)}{2\sigma^2} (1 + 2\rho_o \cos\varphi + \rho_o^2) \right] \right\} \tag{31}$$

This demonstrates the existence of a fading bias that can occur even at large SNR when the interference signal occurs out of phase with the target and tends to cancel its energy. From Table 1 we conclude that the bias due to fading will be negligible as long as the effective SNR is greater than 8 dB. This will be the case if

$$\frac{A_s^2 G_1^2(\theta_s)}{2\sigma^2}\left(1 + 2\rho_o \cos\varphi + \rho_o^2\right) \geq 6.3 \tag{32}$$

For a 20 dB SNR, this shows that fading could become a problem only if $.75 \leq \rho_o \leq 1.25$.

(c) Case 3: Azimuthal Bias

Next, we consider the case in which the SNR is large and the interference is of low enough power that the exponential term in (28) is negligible. Then (29) reduces to

$$\bar{\hat{\theta}} = \frac{\theta_s + (\theta_s + \tilde{\theta}_I)\rho_o \cos\varphi + \rho_o^2 \tilde{\theta}_I}{1 + 2\rho_o \cos\varphi + \rho_o^2} \tag{33}$$

This result is sufficiently general to describe the cases of mainbeam and sidelobe interference. Notice that when $\theta_s = \theta_I = \tilde{\theta}_I$ then $\bar{\hat{\theta}} = \theta_s$ and the estimator is unbiased. Therefore a bias is obtained only when $\theta_s \neq \tilde{\theta}_I$, namely when the target and interference signals are separated in azimuth. It is for this reason that we refer to this effect as the azimuthal bias.

To obtain some physical appreciation for the behaviour of the estimator we consider a specific case of interest. We assume the target is located on boresight at 20 dB SNR. Therefore, $\theta_s = 0$ and $A_s^2 G_1^2(0)/2\sigma^2 = 100$. We next let the interferer be located at the edge of the dB beamwidth so that $\tilde{\theta}_I = \theta_I = .5$. To within a good approximation the sum beam is quadratic over the beamwidth, hence $G_1(\theta) = 1 - 1.7\theta^2$. Since the output interference-to-signal ratio is $\rho_o = \rho G_1(\theta_I)/G_1(\theta_s)$ where $\rho = A_I/A_s$, we see that $\rho_o = .707\rho$. Equation (33) is plotted as a function of the relative phase angle for various values of ρ. The results are shown in Figure 2. Equation (28) was also plotted but there was no discernable difference in the resulting curves at least for the values of ρ we used. In Figure 3 these equations were again plotted for ρ_o nearer to unity and the fading effect can be observed at the out of phase condition. Since there is only a small range of values for which this effect is observable and since its effect is, if anything, beneficial, we shall neglect the fading bias in the rest of our work and restrict our attention to the azimuthal error as described by (29).

The <u>bias</u> in the estimate can be written as

$$b(\varphi) \triangleq \bar{\hat{\theta}} - \theta_s = (\tilde{\theta}_I - \theta_s)\,\rho_o \,\frac{\rho_o + \cos\varphi}{1 + 2\rho_o \cos\varphi + \rho_o^2} \tag{34}$$

where ρ_o and θ_I are given by (23) and (27) respectively. Therefore for small values of ρ_o, $\bar{\hat{\theta}} = \theta_s$, while for large values $\bar{\hat{\theta}} = \theta_I$ which shows how the interferer "captures" the estimate. For intermediate values, curves like those shown in Figures 2 and 3 are obtained which demonstrate the so-called scintillation effect which is a term used to describe the fact that the azimuth estimate lies outside the azimuth interval (θ_s, θ_I).

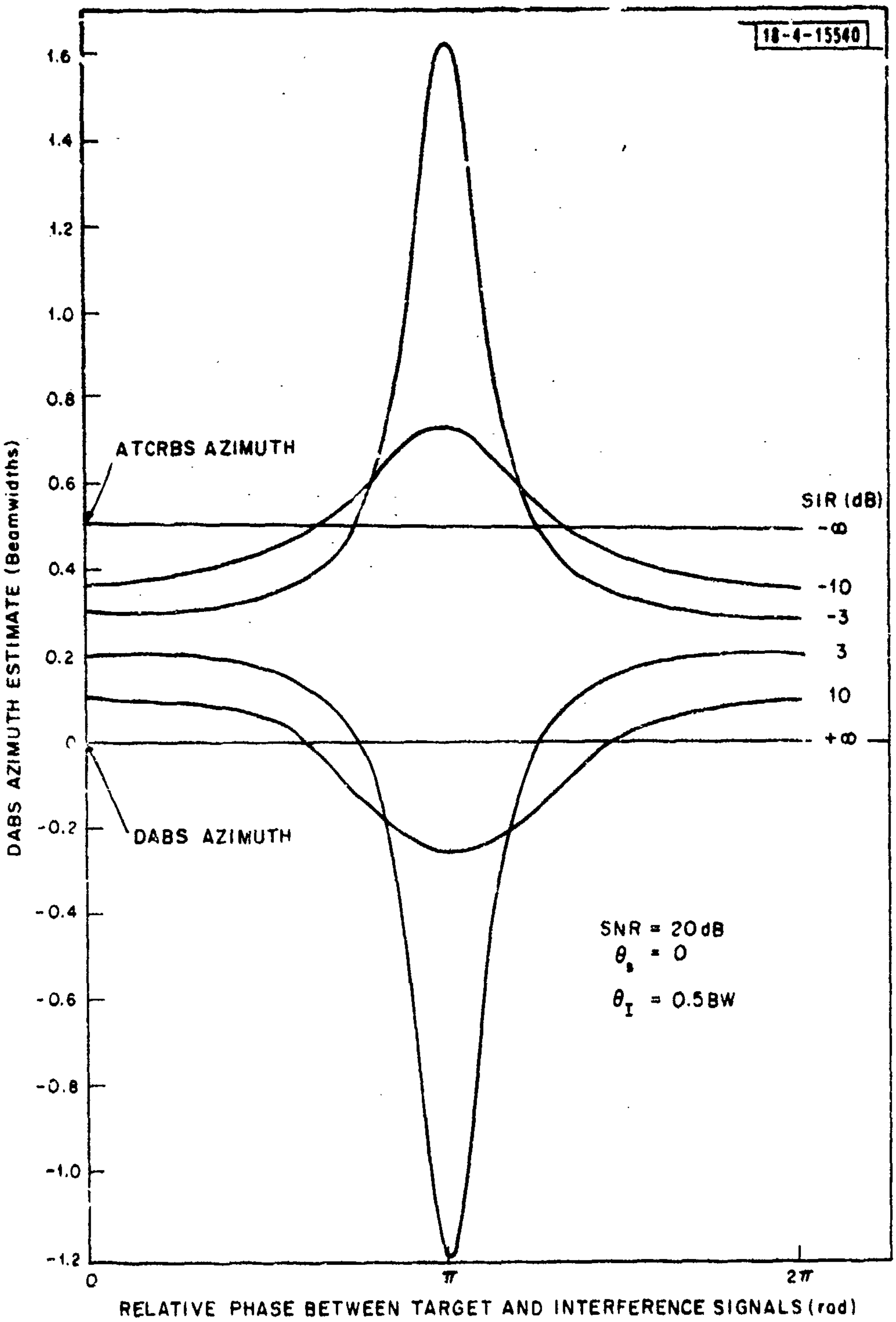

Fig. 2. Azimuth estimate in the presence of interference.

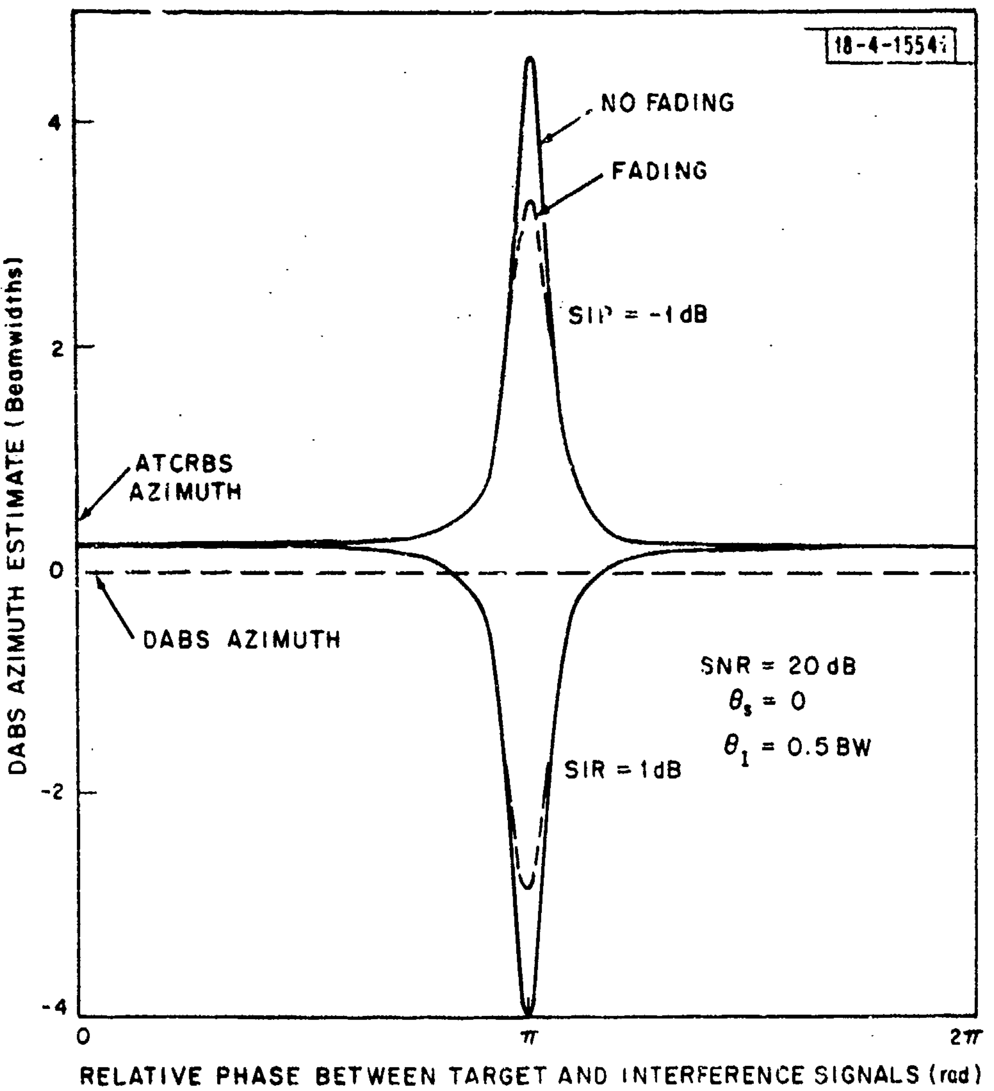

Fig. 3. Azimuth estimate in the presence of interference.

(d) **Case 4: Effects of Relative Phase**

It is reasonable to model the phase as a uniformly distributed random variable on (0, 2π). Then averaging over phase, it is easy to show that

$$\frac{1}{2\pi} \int_0^{2\pi} b(\varphi)\, d\varphi = \begin{cases} 0 & \text{if } \rho_o < 1 \\ 1 & \text{if } \rho_o > 1 \end{cases} \qquad (35)$$

In a practical situation, the key issue is how correlated the phase is from sample to sample within a reply. For multipath, the correlation time can be several seconds because the phase relationship depends on the relative path lengths between the direct and multipath signals and for typical aircraft speeds, these change slowly. For ATCRBS interference, the phase difference will be independent from sample to sample since the transmitter tube is incoherent from pulse to pulse. However, since there are relatively few interference samples in any one reply, the inherent averaging due to phase cannot reliably be exploited and we must face the possibility of having to deal with the large errors shown in Figure 2.

(e) Case 5: Sidelobe Interference

Although all of the results given so far are applicable to mainbeam and sidelobe interference, there are some remarks that are worth noting for the latter case. In the case of ATCRBS sidelobe interference, it is possible that A_I/A_s can be much larger than unity. Let us suppose for example that the DABS target is at maximum range (100 mi) and the ATCRBS interferer is at the same power, but at a close in range (10 mi say). Then the 20 dB decrease in ISR due to the antenna sidelobes is compensated by the 20 dB increase in power due to the range difference. Then ρ_o can be near 0 dB and since the equivalent azimuth is of the order of 1 or 2 beamwidths then, as we have seen, large bias errors can be expected. Therefore we conclude that if a strong ATCRBS signal is being received in a near-in sidelobe, the monopulse azimuth estimate can have a large bias that exhibits the same dependence on the relative phase between the DABS and ATCRBS transponders that was shown to exist for mainbeam interference. Furthermore, it is clear that very low sidelobes is not a sufficient means of eliminating the effects of this interference. Therefore, at least one additional level of data processing will be needed to improve the quality of the azimuth estimate for these cases. This is referred to as monopulse data editing and will be discussed in a later paper [4].

For sidelobe multipath, on the other hand, we can reasonably expect that the reflection coefficient, $A_I/A_s \sim .707$. For antenna patterns with 20 dB peak sidelobe levels, this puts ρ_o less than .07. From our studies so far we know that the bias error peaks when the direct and multipath signals are π out of phase. Using (34) the bias error can be bounded by

$$|b(\varphi)| \leq |b(\pi)| = \frac{\rho_o}{1-\rho_o}|\tilde{\theta}_I - \theta_s| \sim \rho_o|\tilde{\theta}_I - \theta_s| \tag{36}$$

where the last approximation follows from the fact that ρ_o is quite a bit less than unity. For a DABS target on boresight, $\theta_s = 0$ and then using (23) and (27) the bound becomes

$$|b(\varphi)| \leq \frac{1}{k} \frac{A_I}{A_s} \frac{G_2(\theta_I)}{G_1(\theta_s)} \tag{37}$$

which shows that the bias error depends on the sidelobe level of the difference pattern relative to the sum beam gain. For the example studied, $k = 1.5$, $A_I/A_s = .707$ and $G_2(\theta_I)/G_1(\theta_s) = .1$, which yields a peak bias error .047 3 dB beamwidths.

4. INTERFERENCE EFFECTS ON ESTIMATE VARIANCE

In this section we will compute the variance in the monopulse azimuth estimate. Sharenson [3] has studied this problem for the case when the interference consisted only of receiver noise. We shall extend his analysis to include the effects of mainbeam and sidelobe interference. We begin with the equivalent signal model formulated in (1), (2), and (5). The monopulse estimator of interest is

$$\hat{\theta} = \frac{1}{k} \frac{\mathrm{Re}(y_1 y_2^*)}{|y_1|^2} \tag{38}$$

which gives the azimuth estimate in 3 dB beamwidths. From (1) and (6),

$$y_i = \mu_i + n_i \tag{39}$$

so that

$$|y_1|^2 = |\mu_1|^2 \left|1 + \frac{n_1}{\mu_1}\right|^2 \approx |\mu_1|^2 \tag{40}$$

where the last approximation holds provided the equivalent SNR, $|\mu_1|^2/2\sigma^2$ is large enough, typically ≥ 12 dB. Then the moments of (38) can be computed by evaluating the moments of

$$z = \mathrm{Re}(y_1 y_2^*) \tag{41}$$

Since the noise terms n_1 and n_2 are independent, the first moment is simply

$$\bar{z} = \mathrm{Re}\left(\mu_1 \mu_2^*\right) \tag{42}$$

and hence the first moment of the azimuth estimate is given approximately by

$$\bar{\hat{\theta}} = \frac{1}{k} \frac{V_2}{V_1} \cos(\beta_2 - \beta_1) \tag{43}$$

which is the same result we obtained in the last section when the effective SNR was greater than 8 dB. Proceeding further, the second moment of (41) can be calculated using the identity $\mathrm{Re}(x)\mathrm{Re}(y) = \frac{1}{2}\mathrm{Re}(x y^*) + \frac{1}{2}\mathrm{Re}(x y)$. In this case we let $x = y = y_1 y_2^*$ and then

$$\overline{z^2} = \frac{1}{2}\overline{|y_1|^2 |y_2|^2} + \frac{1}{2}\mathrm{Re}\,\overline{y_1^2}\,\overline{y_2^{*2}} \tag{44}$$

From (39) and using the fact that $\overline{|n_i|^2} = 2\sigma^2$, $\overline{n_i^2} = 0$, we obtain

$$\overline{|y_i|^2} = |\mu_i|^2 + 2\sigma^4 \tag{45a}$$

$$\overline{y_i^2} = \mu_i^2 \tag{45b}$$

Combining (42), (44) and (45) we can show that the variance of z is

$$\overline{z^2} - \bar{z}^2 = \left(|\mu_1|^2 + |\mu_2|^2\right)\sigma^2 + 2\sigma^4 \tag{46}$$

Finally we use (40), (46) and (38) to show that the variance of the azimuth estimate is

$$\mathrm{Var}(\hat{\theta}) = \frac{1}{k^2}\frac{\sigma^2}{|\mu_1|^2}\left[1 + \frac{|\mu_2|^2}{|\mu_1|^2} + \frac{2\sigma^2}{|\mu_1|^2}\right] \tag{47}$$

Since $|u_i| = V_i$, then (47) could be expressed in terms of the complex antenna pattern parameters by using (14). Unfortunately the resulting expressions are complicated and in order to obtain some meaningful analytical statements it is not fruitful to retain the most general equations. Therefore, we shall follow the direction of the preceeding section and restrict our attention to the case of real antenna patterns. In addition we note that (47) describes the estimate variance only when the effective SNR is large. This means that $|u_1|^2/2\sigma^2 \gg 1$, hence we can also neglect the effects of the second order SNR term that appears in (47). Under these conditions we can then use (14) for real antenna patterns and show that the variance reduces to

$$\mathrm{Var}(\hat{\theta}) = \frac{\sigma^2}{A_s^2 G_1^2(\theta_s)} \cdot \frac{1 + k^2\theta^2}{k^2} \cdot \frac{1 + 2\rho_o \dfrac{1 + k^2\theta_s\tilde{\theta}_I}{1 + k^2\theta_s^2}\cos\varphi + \rho_o^2 \dfrac{1 + k^2\tilde{\theta}_I^2}{1 + k^2\theta_s^2}}{1 + 2\rho_o\cos\varphi + \rho_o^2} \tag{48}$$

where, as in the case of the bias effects, ρ_o and $\tilde{\theta}_I$ are given by (23) and (27).

In the next few paragraphs we shall consider some more specialized cases.

(a) Case 1: No Interference

When there is no interference, $A_I = 0$, hence $\rho_o = 0$ and from (48) we see that the variance is simply

$$\mathrm{Var}(\hat{\theta}) = \frac{\sigma^2}{A_s^2 G_1^2(\theta_s)} \cdot \frac{1 + k^2\theta_s^2}{k^2} \tag{49}$$

which is the result obtained by Sharenson [3] and others, and shows that the variance decreases with SNR and the slope of the normalized difference pattern and increases with the degree to which the target is off boresight.

(b) Case 2: Fading

If the target and interference arrive from the same azimuth (multipath from a flat earth for example) then $\tilde{\theta}_I = \theta_I = \theta_s$ and (48) becomes

$$\mathrm{Var}(\hat{\theta}) = \frac{\sigma^2}{A_s^2 G_1^2(\theta_s)} \cdot \frac{1 + k^2\theta_s^2}{k^2} \cdot \frac{1}{1 + 2\rho_o \cos\varphi + \rho_o^2} \tag{50}$$

which shows that when multipath fades occur there can be a reduction in the effective SNR which in turn leads to an increase in the variance. In the last section we found that a bias error was also introduced in the fading situation.

(c) Case 3: Azimuthal Variance

When the target and interferer are at different azimuths we see from (46) that the variance will be further increased. Typical results are plotted in Figure 4 for the case of a target on boresight at 20 dB SNR and an interferer located at the edge of the 3 dB beamwidth. We see that the

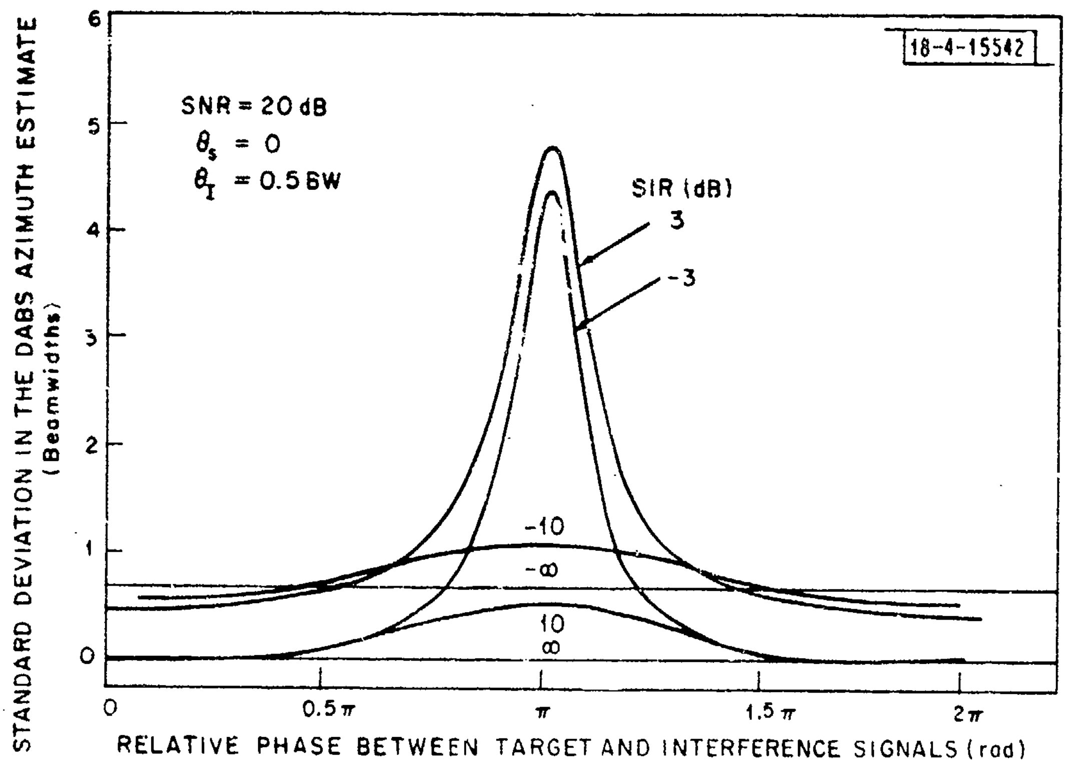

Fig. 4. Variance of the azimuth estimate in the presence of interference.

most significant degradation in performance occurs at the out-of-phase condition. However, the magnitude of the errors is much smaller than the contribution due to the bias, and furthermore since the errors are random, their effect can be further reduced simply by averaging many of the estimates. Therefore, we conclude that the bias error is likely to be the more troublesome problem for ATC direction finding.

5. CONCLUSIONS

An exact expression for the bias of a two beam monopulse azimuth estimate has been derived that describes the degradation in performance due to receiver noise and ATCRBS and multipath interference. At low SNR and high ISR the bias tends toward boresight, although for all practical purposes the net effect of this bias is negligible. More important is the azimuthal bias that describes the so-called scintillation of the azimuth estimate that has been observed in many monopulse tracking problems. Depending on the values of the SNR, ISR and the relative phase between the DABS and interference signals, the bias can be quite significant even when the interfering signal arrives through a low level sidelobe.

A first order analysis was used to obtain the variance in the azimuth estimate when interference is present in a noisy background. Although the results indicate that the random errors will not be insignificant, the deterministic bias error will be, by far, the more dominant effect.

For ISR's less than -30 dB, the effect of the interference is negligible. As this quantity increases, the bias and variance increase and errors that are of the order of a beamwidth can be obtained. For ISR's between $\pm$ 10 dB there is a strong dependence on the relative phase, with a peak error occurring at the out-of-phase condition. As the ISR is further increased, this peaking effect subsides until the interference completely captures the monopulse processor which at this point would track the interference target.

Although the results are applicable to analyzing the effects of ATCRBS interference and multipath, a distinction between the two phenomena should be noted. For multipath, the direct and indirect signals are at the same frequency and coherent in the sense that their relative phase may be constant during several seconds duration. For ATCRBS interference, however, the transponders are incoherent from bit to bit and possibly for samples within a bit since there may be carrier frequency offset. Therefore, in the ATCRBS case, there will be averaging that can be exploited to reduce the overall bias error, in which case the effect of the increased error variance would become a more important effect. From a processing point of view, this could be overcome by using more samples to form the azimuth estimate.

In a separate study [4] the performance of the maximum likelihood interference detector has been presented in detail. The next step is to combine the results of both studies to evaluate the data editing concept. This will determine whether there is any promise to the idea of introducing an additional level of data processing to improve the overall quality of the azimuth estimate.

REFERENCES

1. D.H. Pruslin, "Multipath-Pulse Monopulse Accuracy," Lincoln Laboratory Technical Report 487, (19 November 1971). DDC AD-737369.

2. A.A.L. Browne, Lincoln Laboratory Internal Memorandum.

3. S. Sharenson, "Angle Estimination Accuracy with a Monopulse Radar in the Search Mode, "IRE Transactions on Aerospace and Navigational Electronics (September 1962), pp. 175-179.

4. R.J. McAulay and T.P. McGarty, "Maximum Likelihood Detection of Unresolved Radar Targets and Multipath," to be published.

5. D.K. Barton, "Radar System Analysis, "Prentice-Hall, New Jersey, 1964.

APPENDIX

To compute the average of (9) with respect to α_1, a uniformly distributed random variable on $(0, 2\pi)$ we have

$$E_{\alpha_1\alpha_2}(k\hat{0}) = \cos(\beta_2 - \beta_1)\left\{\frac{1}{2\pi}\int_0^{2\pi}\frac{V_1^2 + V_1N_1\cos\alpha_1}{V_1^2 + N_1^2 + 2V_1N_1\cos\alpha_1}\,d\alpha_1\right\}$$

$$+ \sin(\beta_2 - \beta_1)\left\{\frac{1}{2\pi}\,\frac{V_2}{V_1}\int_0^{2\pi}\frac{V_1N_1\sin\alpha_1}{V_1^2 + N_1^2 + 2V_1N_1\cos\alpha_1}\,d\alpha_1\right\}$$

Since the integrand of the second term is an odd function, its integral is zero. Using standard integral tables, Browne showed that the integral in the first term reduces to

$$\cos(\beta_2 - \beta_1)\,\frac{1}{4\pi}\,\frac{V_2}{V_1}\left\{2\pi + \frac{V_1^2 - N_1^2}{V_1^2 + N_1^2}\int_0^{2\pi}\frac{1}{\left(1 + \frac{2V_1N_1}{V_1^2 + N_1^2}\cos\alpha_1\right)}\,d\alpha_1\right\}$$

$$= \frac{1}{2}\,\frac{V_2}{V_1}\cos(\beta_2 - \beta_1)\left\{1 + \frac{V_1^2 - N_1^2}{V_1^2 + N_1^2}\cdot\left[1 \quad \frac{4V_1^2N_1^2}{(V_1^2 + N_1^2)^2}\right]^{1/2}\right\}$$

where the positive square root is understood. Therefore

$$E_{\alpha_2\alpha_1}(k\hat{0}) = \begin{cases} 0 & \text{if } N_1 < V_1 \\ \dfrac{V_2}{V_1}\cos(\beta_2 - \beta_1) & \text{if } N_1 > V_1 \end{cases}$$

as given in (10).

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